

# Impact of the bounds on Higgs mass and $m_W$ on effective theories

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## Abstract

We study the inter-relations that exist between the present experimental bounds on the Higgs mass, as obtained from radiative corrections to  $m_W$ , and the effective parameters,  $\alpha_i$  and  $\Lambda$ . We find that the SM bounds on  $m_H$ , arising from a precise determination of the  $W$  mass, can be substantially modified by the presence of dimension-6 operators which appear in the linear realization of the effective Lagrangian approach. A Higgs mass as heavy as 700 GeV can be allowed for scales of new physics of the order of 1 TeV.

## 1 Introduction

Some standard model (SM) parameters have been measured with such a high precision that has allowed to constrain the values of other SM parameters, or even new physics, through the use of radiative corrections[1], as can be exemplified by the correct agreement between the predicted top mass and the observed value [2]. Finding the Higgs boson remains as the final step to confirm the theoretical scheme of the SM. The present lowest experimental bound on the Higgs mass is  $m_H > 90.4$  GeV [3], this is a direct search limit. On contrary to the top quark case, radiative correction are only logarithmically sensitive to the Higgs mass, and thus it is more difficult to obtain an indirect bound. However, fits with present data seems to favor a light SM Higgs [4]. Henceforth, it is interesting to ask how this conclusion will change if one goes beyond the SM.

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The framework of effective Lagrangians, as a mean to parametrize physics beyond the SM in a model independent manner, has been used extensively recently [5]. Within this approach, the effective lagrangian is constructed by assuming that the virtual effects of new physics modify the SM interactions, and these effects are parametrized by a series of higher-dimensional nonrenormalizable operators written in terms of the SM fields. The effective linear Lagrangian can be expanded as follows:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{i,n} \frac{\alpha_i}{\Lambda^n} O_n^i \quad (1)$$

where  $\mathcal{L}_{\text{SM}}$  denotes the SM renormalizable lagrangian. The terms  $O_n^i$  are  $SU(3) \times SU(2)_L \times U(1)_Y$  invariant operators.  $\Lambda$  is the onset scale where the appearance of new physics will happen. The parameters  $\alpha_i$  are unknown in this framework, although "calculable" within a specific *full theory* [6]. This fact was used in Ref. [7] to show that, in a weakly coupled full theory, a hierarchy between operators arises by analyzing the order of perturbation theory at which each operator could be generated *e.g.* by integrating the heavy degrees of freedom. Some operators can be generated at tree-level, and it is natural to assume that their coefficients will be suppressed only by products of coupling constants; whereas the ones that can be generated at the 1-loop level, or higher, will be also suppressed by the typical  $1/16\pi^2$  loop factors. This allows us to focus on the most important effects the high-energy theories could induce, namely those coming from tree-level generated dimension-six operators.

In this letter we address two related questions. First, we study how the effective lagrangian affects the determination of the W boson mass, and how this could affect the bounds on the Higgs mass. The second item under consideration will be to re-examine the effects on Higgs-vector boson production at hadron colliders by using the results obtained from the first part.

## 2 The SM $W$ mass

We shall use the results of Ref. [8,9], which parametrize the bulk of the radiative corrections to  $W$  mass through the following expression:

$$m_W = m_W^o [1 + d_1 \ln(\frac{M_h}{100}) + d_2 C_{em} + d_3 C_{top} + d_4 C_{as} + d_5 \ln^2(\frac{M_h}{100})], \quad (2)$$

where the coefficients  $d_i$  are given in table 2 of Ref. [9], they incorporate the full 1-loop effects, and some dominant 2-loop corrections. The factors  $C_i$  are given by:

$$\begin{aligned}
C_{em} &= \frac{\Delta\alpha_h}{0.0280} - 1, \\
C_{top} &= \left(\frac{m_t}{175\text{GeV}}\right)^2 - 1, \\
C_{as} &= \frac{\alpha_s(M_Z)}{0.118} - 1,
\end{aligned} \tag{3}$$

and they measure the dependence on the fine structure constant, top mass and strong coupling constant, respectively. The reference  $W$  mass,  $m_W^0 = 80.383$  GeV, is obtained with the following values:  $m_t = 175$  GeV,  $\alpha_s = .118$ ,  $\Delta\alpha = 0.0280$ , and  $m_H = 100$  GeV. By using eqs. (2-3), one finds a bound on the SM Higgs mass,  $m_t = 176 \pm 2$  GeV,  $\Delta\alpha = .0280$  and  $\alpha_s = .118$ , of  $170 < m_H < 330$  GeV. This result agree with several other studies [1] which anticipates the existence of a light Higgs boson.

Including the effects of new physics will modify the  $W$  mass value. That effect should be combined with the previous SM corrections in order to determine to what extent new physics could change the bounds obtained for the Higgs mass.

### 3 Modification to the $W$ mass

The complete set of effective operators is large but the analysis simplifies because loop-level dimension six operators and tree-level dimension eight ones generate subdominant effects with respect to tree-level generated dimension six effective operators <sup>2</sup>. The effective contributions to the input parameters in the formulas (2-3), besides  $m_W$ , can be show to disappear after suitable redefinitions [11]. Just two operators contribute:

$$\begin{aligned}
O_\phi^{(1)} &= (\phi^\dagger\phi)(D^\mu\phi^\dagger D_\mu\phi), \\
O_\phi^{(3)} &= (\phi^\dagger D_\mu\phi)(D^\mu\phi)^\dagger\phi.
\end{aligned} \tag{4}$$

The notation is as usual,  $\phi$  denotes the Higgs doublet, and  $D_\mu$  is the usual covariant derivative. An interesting characteristic of these operators is that only  $O_\phi^{(1)}$  gives contribution to  $m_W$ ; although both operators modify the coefficients of the vertices  $HVV$  they leave intact its Lorentz structure. This approach is equivalent to the most usual one used in the literature in which more operators are allowed but they are constrained in a one-per-one basis, so that "unnatural" cancellations are not allowed.

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<sup>2</sup> This is not the case when tree-level dimension six operators do not contribute [10,11].

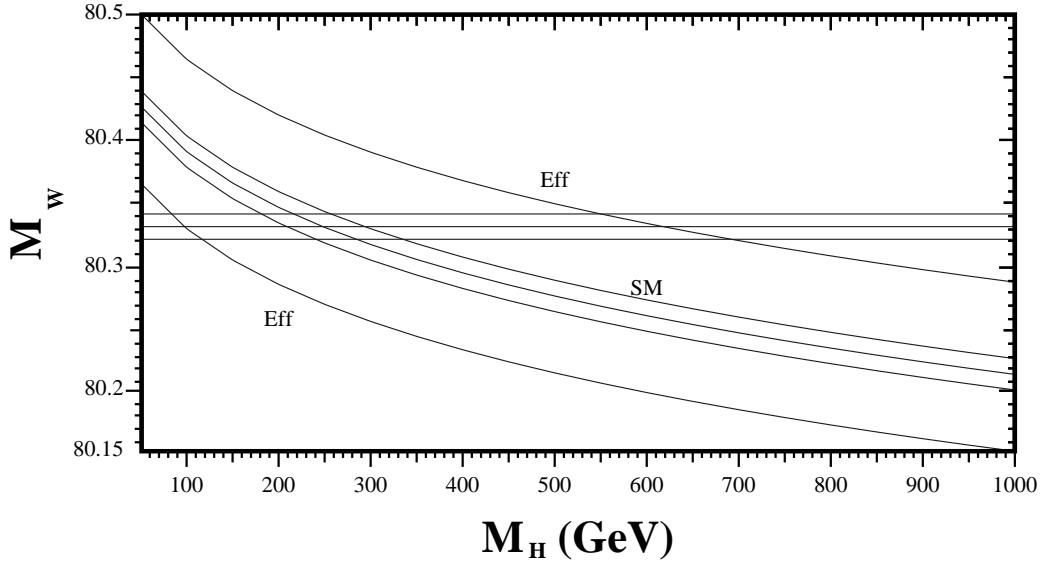


Fig. 1.  $m_W$  versus  $m_H$  for  $m_t = 176 \pm 2$  GeV, both in the Standard Model (SM) and in the maximum effective contributions (Eff).

We shall now include the contribution to  $m_W$  arising from the effective operators of eq. (4) into eqs. (2). The formulae for the  $W$  mass becomes:

$$m_W|_{\text{eff}} = m_W|_{\text{SM}} \left(1 + \frac{1}{4} \alpha_\phi^{(1)} \left(\frac{v}{\Lambda}\right)^2\right) \quad (5)$$

where  $m_W|_{\text{SM}}$  corresponds to the  $W$  mass as defined in eq. (2), and the term in the parentheses arises from the effective Lagrangian.

To study the inter-relations between the Higgs mass and  $\alpha_\phi^{(1)}$  and  $\Lambda$ , we must set the allowed  $m_W - m_H$  region. We are going to use the future expected minimum uncertainty for the  $W$  mass:  $\Delta m_W = \pm .01$  GeV for a nominal central value of  $m_W = 80.33$  GeV, and expand  $m_H$  between the recent experimental bound,  $m_H \geq 90$  GeV [3], and the perturbative limit,  $m_H \leq 700$  GeV (Fig. 1). We take an optimum scenarios also for the top quark mass ( $m_t = 176 \pm 2$  GeV).  $\alpha$  and  $\Lambda$  should take values such that the resulting masses must satisfy the above constraints. Since we assumed the effective Lagrangian is derived from a weakly coupled theory, it is reasonable to impose<sup>3</sup>  $|\alpha_\phi^{(1)}| \leq 1$ , while  $\Lambda$  is set to values greater or equal than 1 TeV. This scale is a conventional one, it can be justified in some specific high-energy models. It turns out that the case when  $|\alpha_\phi^{(1)}| = 1$  and  $\Lambda = 1$  TeV, simultaneously, is already excluded by present data. Figure 2 shows the level curves in the  $\alpha - \Lambda$  plane according with the

<sup>3</sup> We will choose the  $\alpha_i$  signs that give the maximum and minimum values for the quantities of interest.

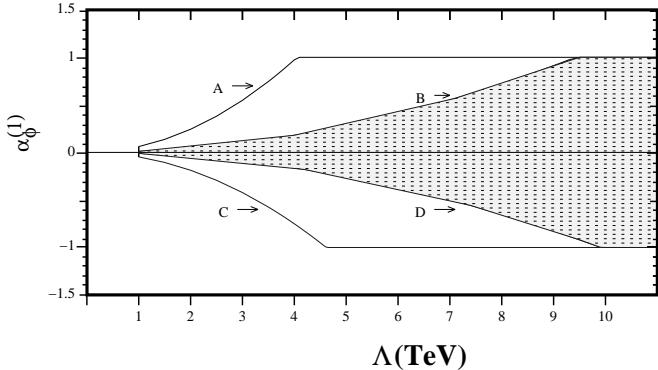


Fig. 2. Constrained region for  $\alpha_\phi^{(1)}$  and  $\Lambda$ .  $m_t = 176$  GeV.

bounds discussed before for  $m_W$  and  $m_H$ . The allowed region of parameters corresponds to the area located to the right of curves A, B, C, D. Allowing for an enlargement of the allowed Higgs mass bound from  $170 < m_H < 330$  GeV in the SM to  $90 < m_H < 700$  GeV into the effective Lagrangian case. The curve A is obtained by taking  $m_H = 700$  GeV and  $m_W = 80.32$  GeV, while for the curve B we use  $m_H = 90$  GeV and  $m_W = 80.34$  GeV. The shadowed area between the curves (BD) marks the parameters region where no new physics effects can be disentangled from the SM uncertainties. These results were obtained by considering  $m_t = 176$  GeV; it is found that there are not substantial changes by adding the top mass uncertainty to the effective contributions. It is also found that the allowed ranges for  $\Lambda$  and  $\alpha_\phi^{(1)}$  are as follow: for  $\Lambda = 1$  TeV,  $.011 < \alpha_\phi^{(1)} < .060$  and  $-.01010 < \alpha_\phi^{(1)} < -.01019$ . This correspond to Higgs masses between the perturbative limit and upper SM bound in the first interval, and between the lower SM bound (obtained from radiative corrections to  $m_W$ ) and the experimental limit for the second interval. In the complementary case, i.e. taking  $\alpha_\phi^{(1)}$  equal to its maximum value, it is found that  $4.1 < \Lambda < 9.5$  TeV, and  $4.64 < \Lambda < 10$  TeV with same considerations for the Higgs mass as in the first case. Then, independently of the value  $\alpha_\phi^{(1)}$  effects arising from a scale beyond of 10 TeV can not be disentangled from SM top uncertainties.

#### 4 Associated $W(Z)$ and $H$ production

In this paper we also re-examined accordingly the modifications to the mechanism of associated production of Higgs boson with a vector particle ( $W, Z$ ), due to the effective Lagrangian updating the results obtained in Ref. [13]. The

corresponding Lagrangian to be used is :

$$\mathcal{L}_{HVV} = \frac{m_Z}{2}(1+f_1)HZ_\mu Z^\mu + gm_W(1+f_2)HW_\mu^+W^{-\mu}, \quad (6)$$

where the parameter  $f_i$  are functions of  $\epsilon_j = \alpha_j(\frac{v}{\Lambda})^2$  given as follows:

$$f_1 = \frac{1}{2}(\epsilon_\phi^{(1)} + \epsilon_\phi^{(3)}), \quad f_2 = \frac{3}{4}(2\epsilon_\phi^{(1)} - \epsilon_\phi^{(3)}). \quad (7)$$

The ratio of the effective cross-section to the SM one for the processes  $p\bar{p} \rightarrow H + V$  have been evaluated. The parton convolution part is factored out, and only remains the ratio of partonic cross-sections, thus the result is valid for both FNAL and LHC. The expressions for the cross-sections ratios are:

$$\begin{aligned} R_{HW} &= \frac{\sigma_{\text{eff}}(p\bar{p} \rightarrow H + W)}{\sigma_{\text{SM}}(p\bar{p} \rightarrow H + W)} \\ &= (1+f_2)^2 \end{aligned} \quad (8)$$

$$\begin{aligned} R_{HZ} &= \frac{\sigma_{\text{eff}}(p\bar{p} \rightarrow H + Z)}{\sigma_{\text{SM}}(p\bar{p} \rightarrow H + Z)} \\ &= (1+f_1)^2 \end{aligned} \quad (9)$$

For the operators under consideration, the cross-sections ratio is independent of the Higgs mass, and as the best values, it is found that the cross-section is only slightly modified. The behavior of the cross-section ratios are shown in fig. 3, for typical values of  $\alpha$  and  $\Lambda$  as found in section 4. We consider the same estimate for  $\alpha_\phi^{(3)}$  as we got for  $\alpha_\phi^{(1)}$ . As it can seen from figure 3, these two processes result almost insensible to new physics effects arising from the dimension-six operators  $O_\phi^{(1,3)}$ , since their effects is of the order  $10^{-3}$ . The corresponding contributions that arises from the effective operators that were neglected are, at least, 2 orders of magnitude below that ones that are considered here. Of course, a more detailed study is needed in order to include the modifications to the expected number of events for discovery as a function of  $m_H$ , and that case is under study.

## 5 Conclusions

We have studied the modifications that new physics imply for the bound on the Higgs mass that is obtained from radiative corrections to electroweak observables, within the context of effective lagrangians. We found that the SM

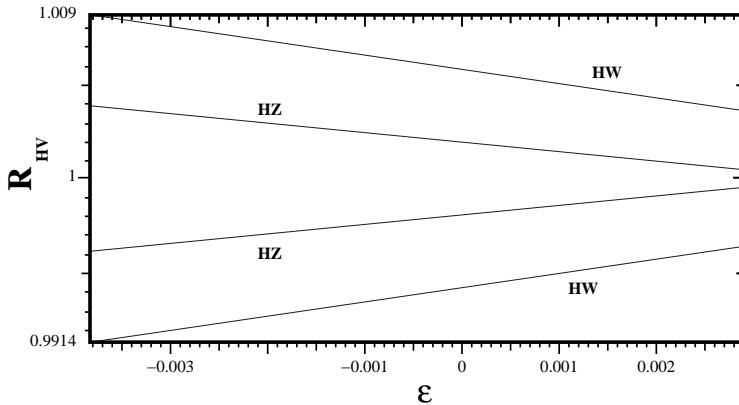


Fig. 3. Ratio of the effective cross section to the SM one for the associated  $HW, HZ$  production. It is assumed that  $\alpha_\phi^{(3)}$  behaves in the same way as  $\alpha_\phi^{(1)}$  does.

bound  $170 \leq m_H \leq 330$  GeV that is obtained from a precise determination of the  $W$  mass, can be substantially modified by the presence of dimension-6 operators that arise in the linear realization of the effective Lagrangian approach. A Higgs mass as heavy as 700 GeV is allowed for scales of new physics of the order of 1 TeV, with a corresponding value for  $|\alpha_\phi^{(1)}|$  of the order of  $10^{-2}$ . As well we found that even for  $|\alpha_\phi^{(1)}| = 1$ , new physics effects arising from scales  $\Lambda > 10$  TeV can not be separate from the uncertainties on the top quark mass, in an optimum scenarios for the observables considered here. Those results give us the landmark for the decoupling limits for both  $\alpha_i$  and  $\Lambda$ . Accordingly, it is found that such operators do not produce a significant modification for the present (FNAL) or future (LHC) studies for the associated production mechanism  $p\bar{p} \rightarrow H + V$ .

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